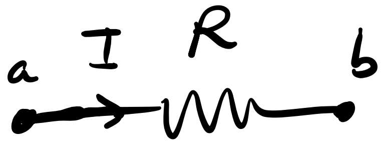
PHYS 231 — Sept. 11, 2023

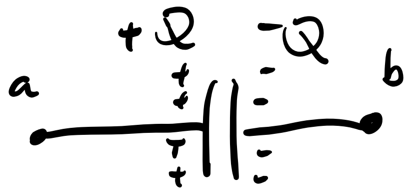
Last Time:

Resistors



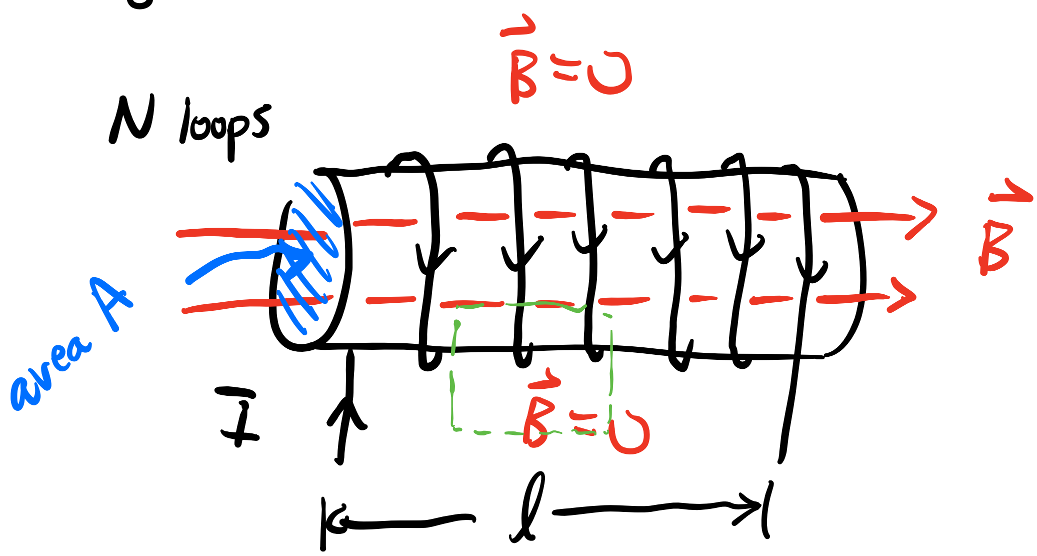
$$\Delta V_R = V_b - V_a = -IR$$

Capacitors



$$\Delta V_C = V_b - V_a = -\frac{Q}{C}$$

Today: Inductors / Solenoids



From Ampère's Law, can calc. magnetic field inside bore of solenoid.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow B = \mu_0 n I$$

$$n = \frac{N}{l}$$

$$\boxed{\therefore B = \mu_0 \frac{N}{l} I}$$

By Faraday's Law, a changing magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$ induces emf $\mathcal{E}$ in the coil.

$$|\mathcal{E}| = \frac{d\Phi_B}{dt}$$

magnetic flux through one loop of inductor is

$$\Phi_{B1} = BA = \mu_0 \frac{N}{l} A I$$

$\therefore$ Since our inductor has N loops, net magnetic flux is: $\Phi_B = N\Phi_{B1} = \mu_0 \frac{N^2}{l} A I$

∴ from Faraday's Law, the voltage across the inductor is

$$|\Delta V_L| = \frac{d\Phi_B}{dt} = \frac{d}{dt}\left(\mu_0 \frac{N^2}{l} A I\right)$$

"inductance"

Everything is constant expect for I.

$$\therefore |\Delta V_L| = \underbrace{\left(\mu_0 \frac{N^2}{l} A\right)}_{\text{inductance } L} \frac{dI}{dt}$$

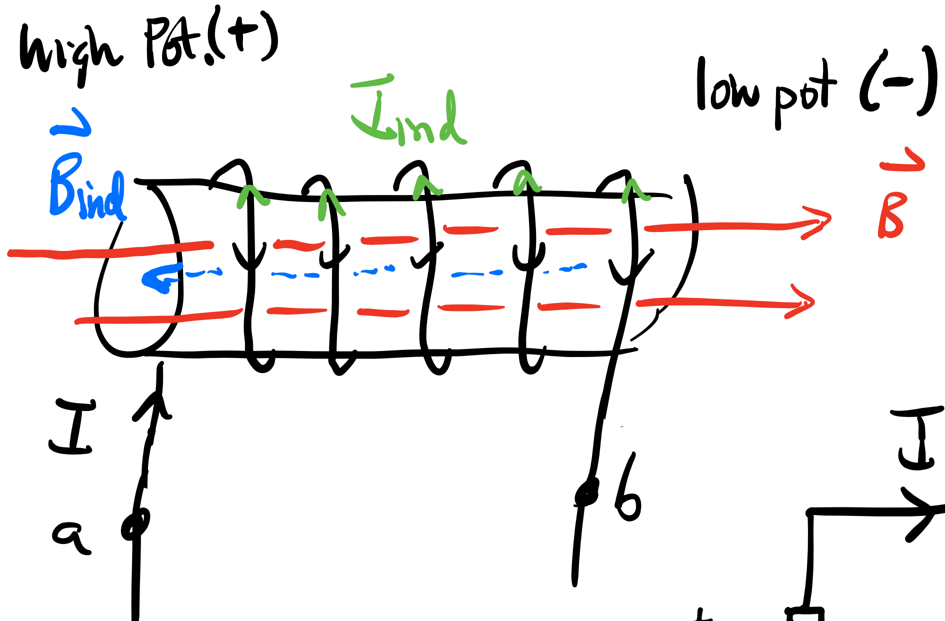
$$\boxed{\therefore |\Delta V_L| = L \frac{dI}{dt}}$$

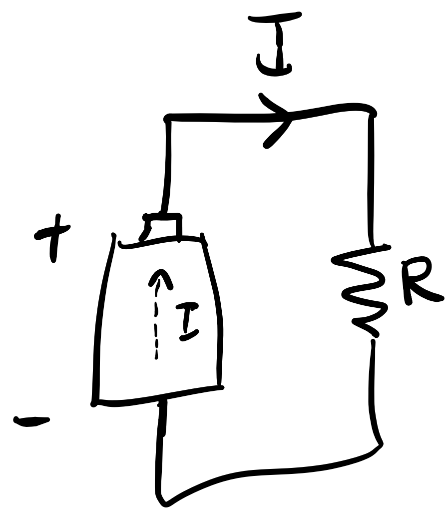
Notice that if I is constant, then $\frac{dI}{dt} = 0$ & $\Delta V_L = 0$.

Inductance opposes the change in current through the coil.

Lenz's law tells the sign of the voltage across an inductor.

Lenz's Law says that induced emfs drive currents that oppose change in magnetic flux.

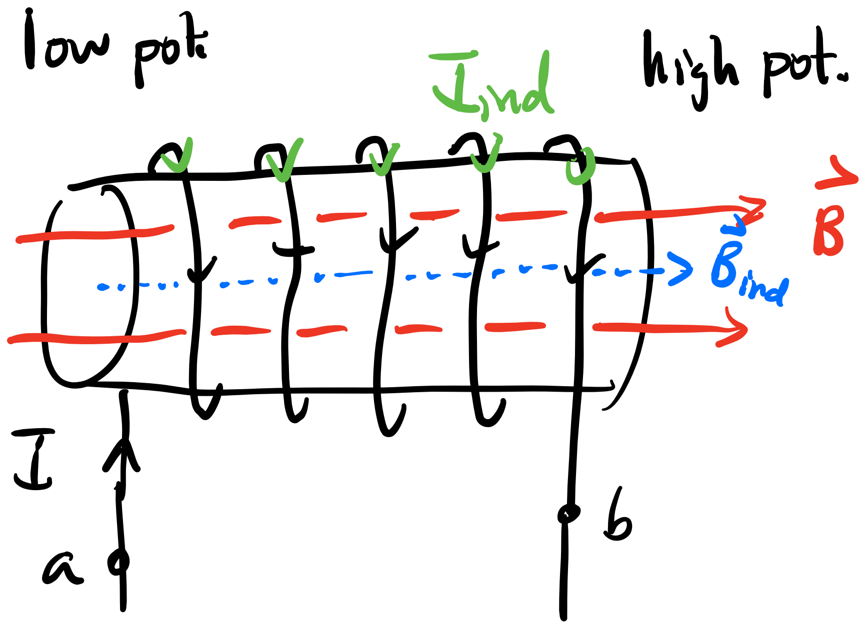




(i) I is increasing.

Then, the voltage difference

$$\Delta V_L = V_b - V_a = -L\frac{dI}{dt} < 0$$



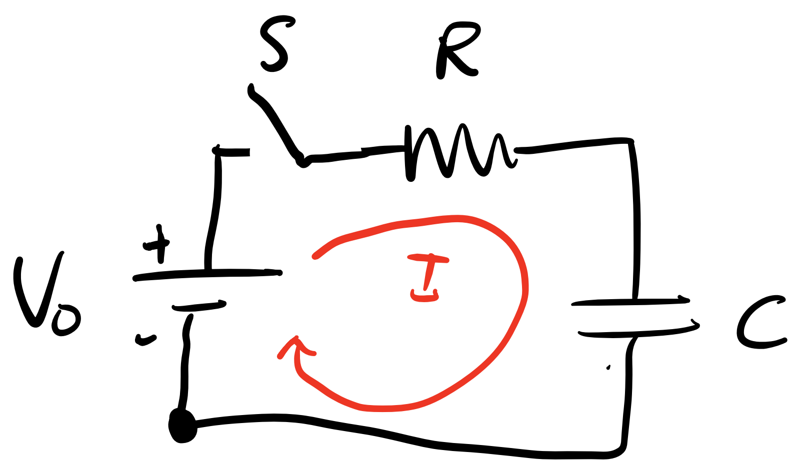
(ii) I decreasing.

$$\Delta V_L = V_b - V_a = -L\underbrace{\frac{dI}{dt}}_{\text{negative since } I \text{ is decreasing.}} > 0$$

Summary: Capacitors, Resistors, Inductors.

$$\boxed{\begin{aligned} |\Delta V_C| &= \frac{Q}{C} \\ |\Delta V_R| &= IR = R\frac{dQ}{dt} \\ |\Delta V_L| &= L\frac{dI}{dt} = L\frac{d^2Q}{dt^2} \end{aligned}}$$

Think about simple circuits involving R, C, L.



Initially switch is open and capacitor is uncharged. Then at $t=0$, switch is closed.

Kirchhoff voltage loop rule: Net change in voltage around a closed loop is zero.

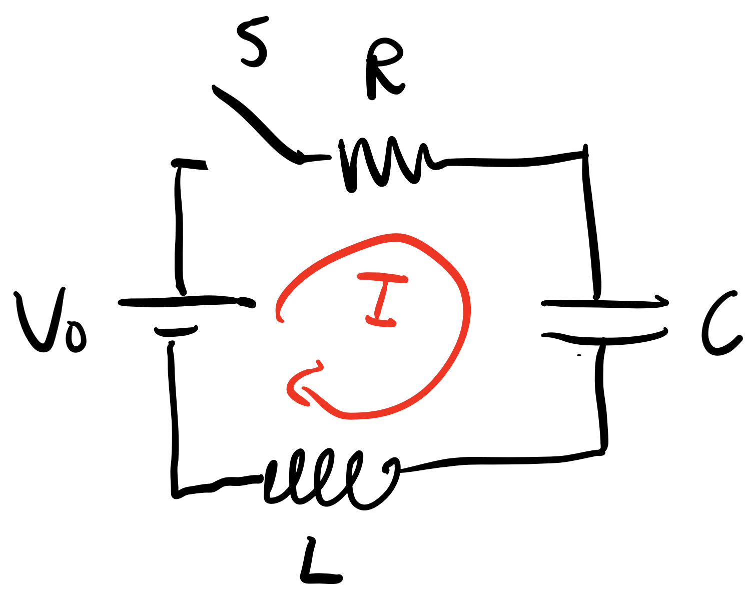
KVL: $$V_0 - \Delta V_R - \Delta V_C = 0$$

$$\therefore V_0 - IR - \frac{Q}{C} = 0$$

or $$\boxed{V_0 = \frac{1}{C} Q + R \frac{dQ}{dt}}$$ (4)

differential eq'n that governs the time evolution of the charge Q in our RC circuit.

Consider a LRC circuit.



switch closed at $t=0$.

Capacitor is initially uncharged.

KVL: $$V_0 - IR - \frac{Q}{C} - L\frac{dI}{dt} = 0$$

(#) $$\boxed{V_0 = \frac{1}{C}Q + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2}}$$

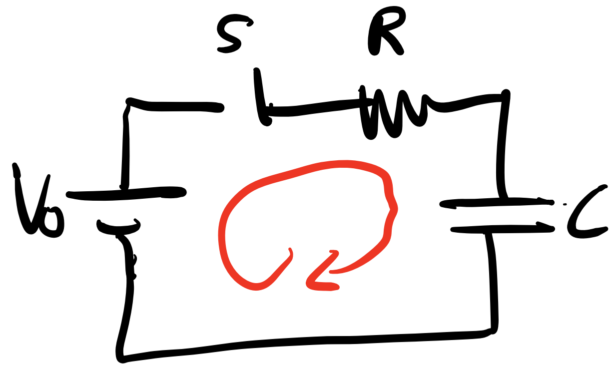
2nd order differential equation for $Q(t)$ in LRC circuit.

We want to solve for the time dependence of $Q(t)$ in eq'ns (4) & (#).

This is called solve for the transient response of the circuit.

Let's see if we can solve ⊛ for $Q(t)$.

$$V_0 = \frac{1}{C} Q + R \frac{dQ}{dt} \qquad ⊛$$



A long time after the switch is closed, expect ① capacitor to be charged &

② $I = \frac{dQ}{dt} = 0$.

If $\frac{dQ}{dt} = 0$, then ⊛ becomes.

$$V_0 = \frac{1}{C} \underbrace{Q_0}_{Q \text{ when } t \to \infty} \implies \underbrace{Q_0 = V_0 C}_{\substack{\text{charge on} \\ \text{capacitor when} \\ t \to \infty.}}$$

$\Delta V_c = \frac{Q}{C}$ when $t \to \infty$ $\Delta V_c = \frac{Q_0}{C}$

$= \frac{V_0 \cancel{C}}{\cancel{C}} = \boxed{V_0}$

As expected when $t \to \infty$

$\Delta V_c =$ battery voltage.